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Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL SENIOR CERTIFICATE/
*NASIONALE SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

**These marking guidelines consist of 24 pages.
*Hierdie nasienriglyne bestaan uit 24 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering</i> <i>('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede</i> <i>('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

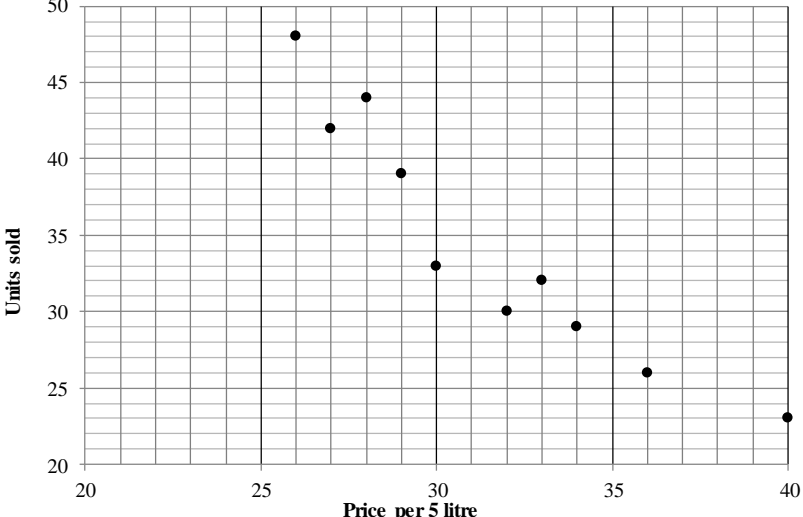
QUESTION/VRAAG 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$	<div>Answer only: Full marks</div> <div>Slegs antw: Volpunte</div>	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10,1707 \approx 10,17$		✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32,17$ $\therefore 5$ days		✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/ <i>bestel</i> $20 \times 18 = 360$ sold/ <i>verkoop</i> Total not sold/ <i>Totaal nie verkoop nie</i> : 36 OR/OF $22 - 20 = 2$ $2 \times 18 = 36$		✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2)
1.3.1	Option B/ <i>Opsie B</i> <u>Any one of the following reasons/Enige een van die vlg redes:</u> <ul style="list-style-type: none"> Median/<i>Mediaan</i> = 18,5 $Q_1 = 14$ IQR = 21 Mean > Median, therefore the data is skewed to the right 		✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right <i>Data is positief skeef/skeef na regs</i>		✓ answer (1)
			[10]

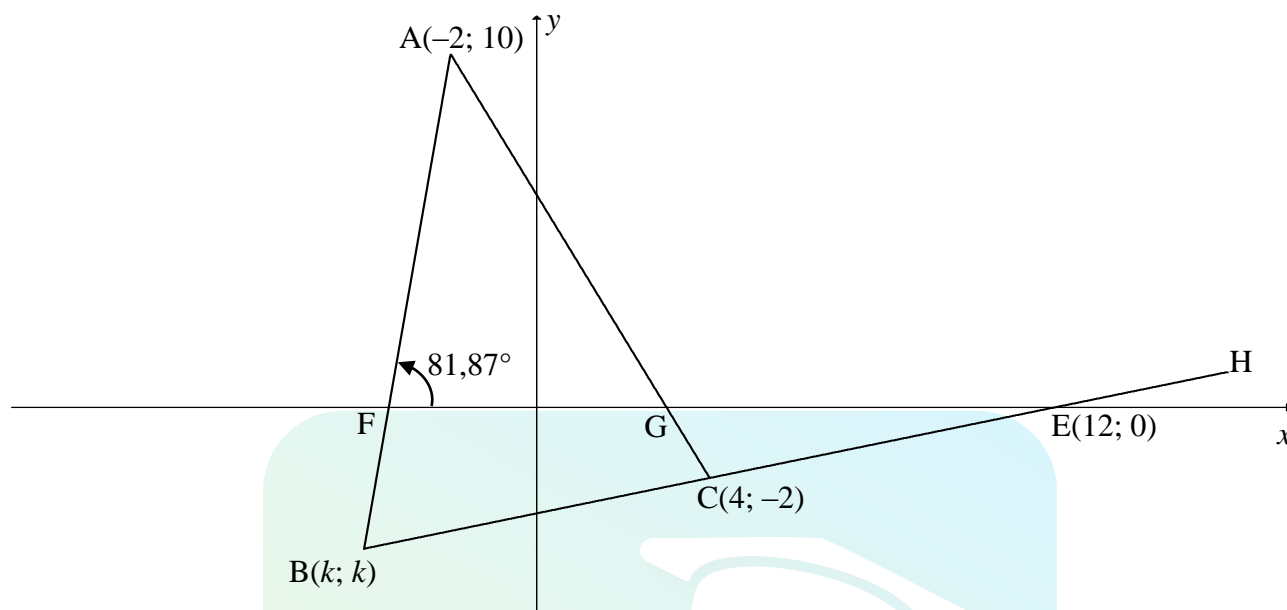
QUESTION/VRAAG 2

Price of milk in rands per 5-litre container (x) <i>Prys van melk in rand, per 5 liter-houer (x)</i>	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) <i>Aantal 5 liter-houers melk verkoop (y)</i>	48	30	26	44	23	32	39	29	42	33

2.1	<p style="text-align: center;">SCATTER PLOT</p> 	<p>1 mark: 3 to 5 points plotted correctly</p> <p>2 marks: 6 to 9 points plotted correctly</p> <p>3 marks: all points plotted correctly</p> <p style="text-align: right;">(3)</p>
2.2	$a = 90,478... \approx 90,48$ $b = -1,773... \approx -1,77$ $\hat{y} = 90,48 - 1,77x$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Answer only: Full marks <i>Slegs antw: Volpunte</i></p> </div>	<p>✓ a ✓ b ✓ equation</p> <p style="text-align: right;">(3)</p>
2.3	$y = 23,069... \approx 23,07$ units/eenhede (calculator/sakrekenaar) OR/OF $y = 90,48 - 1,77(38)$ $y = 23,22$ units/eenhede	<p>✓✓ answer</p> <p style="text-align: right;">(2)</p> <p>✓ substitution ✓ answer</p> <p style="text-align: right;">(2)</p>
2.4	$r = -0,94$ The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold \therefore there is a good chance of the prediction being accurate./ <i>Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan \therefore daar is 'n goeie kans dat die voorspelling akkuraat is</i>	<p>✓ value of r OR/OF strong relationship/ <i>sterk verwantskap</i></p> <p>✓ accurate/akkuraat</p> <p style="text-align: right;">(2)</p>

[10]

QUESTION/VRAAG 3

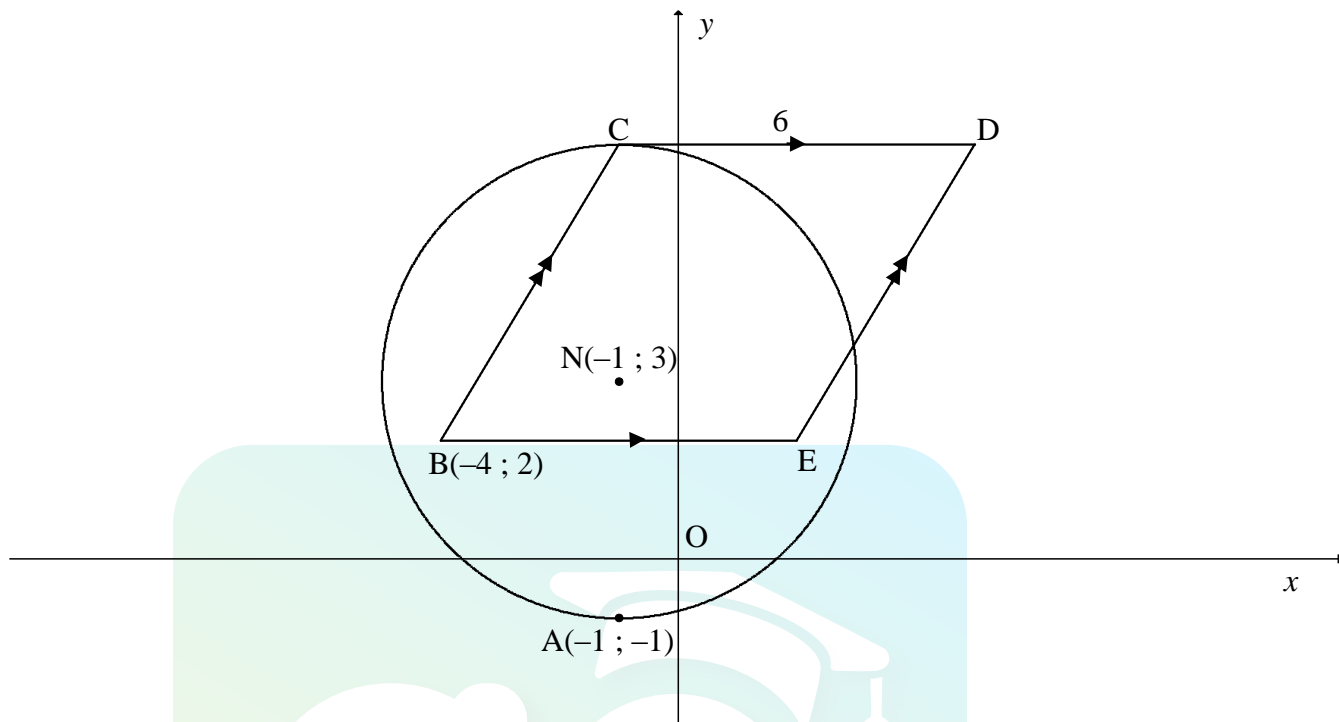


3.1.1	$m_{BE} = m_{CE} = \frac{0 - (-2)}{12 - 4}$ $= \frac{1}{4}$	OR/OF $m_{BE} = m_{CE} = \frac{-2 - 0}{4 - 12}$ $= \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81,87^\circ$ $m_{AB} = 7$	Answer only: Full marks <i>Slegs antw: Volpunte</i>	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ $0 = \frac{1}{4}(12) + c$ $c = -3$ $y = \frac{1}{4}x - 3$ OR/OF $y = mx + c$ $-2 = \frac{1}{4}(4) + c$ $c = -3$ $y = \frac{1}{4}x - 3$	$y - y_1 = m(x - x_1)$ $y - 0 = \frac{1}{4}(x - 12)$ $y = \frac{1}{4}x - 3$ $y - y_1 = m(x - x_1)$ $y - (-2) = \frac{1}{4}(x - 4)$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

3.3.1	$y = \frac{1}{4}x - 3$ $k = \frac{1}{4}k - 3$ $\frac{3}{4}k = -3$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> $m_{BE} = \frac{1}{4}$ $\frac{0 - k}{12 - k} = \frac{1}{4}$ $-4k = 12 - k$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> $m_{AB} = \tan 81,87^\circ$ $m_{AB} = 7$ $m_{AB} = \frac{10 - k}{-2 - k}$ $7(-2 - k) = 10 - k$ $-14 - 7k = 10 - k$ $-6k = 24$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> <p>EB: $y = \frac{1}{4}x - 3$ and AB: $y = 7x + 24$</p> $\frac{1}{4}x - 3 = 7x + 24$ $\frac{27}{4}x = -27$ $x = k = -4$ $\therefore B(-4; -4)$	<p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ equating EB & AB</p> <p>✓ answer (2)</p>
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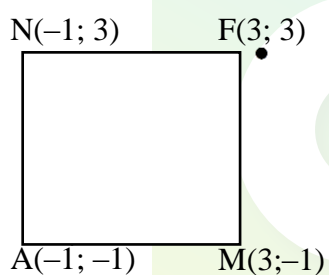
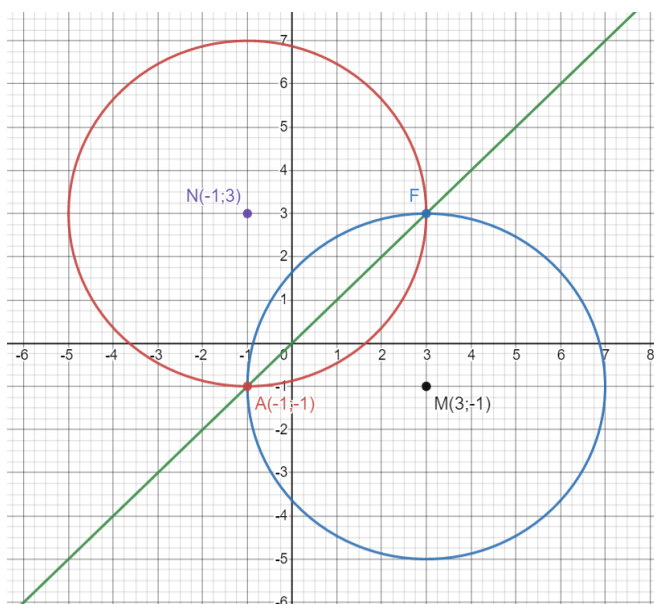
3.3.2	<p>In $\triangle AFG$:</p> $m_{AC} = \frac{10 - (-2)}{-2 - 4} = -2$ $\tan \theta = m_{AC} = -2$ $\theta = 180^\circ - 63,43\dots^\circ$ $\therefore \theta = 116,57^\circ$ $\therefore \hat{A} = 116,57^\circ - 81,87^\circ [\text{ext } \angle \text{ of } \Delta]$ $\therefore \hat{A} = 34,70^\circ$ <p>OR/OF</p> <p>In $\triangle ABC$:</p> $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc \cdot \cos A$ $(2\sqrt{17})^2 = (6\sqrt{5})^2 + (10\sqrt{2})^2 - 2(6\sqrt{5})(10\sqrt{2}) \cdot \cos A$ $\cos A = \frac{(6\sqrt{5})^2 + (10\sqrt{2})^2 - (2\sqrt{17})^2}{2(6\sqrt{5})(10\sqrt{2})}$ $= 0,822\dots$ $\therefore A = 34,7^\circ$	<p>✓ $m_{AC} = -2$</p> <p>✓ $\tan \theta = -2$</p> <p>✓ $\theta = 116,57^\circ$</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
3.3.3	$M\left(\frac{12 + (-2)}{2}; \frac{10 + (0)}{2}\right)$ <p>Diagonals intersect at the point (5 ; 5)</p>	<p>✓ x-value ✓ y-value</p> <p style="text-align: right;">(2)</p>
3.4.1	<p>BE = ET</p> $4\sqrt{17} = \sqrt{(12 - p)^2 + (0 - p)^2}$ $(4\sqrt{17})^2 = (\sqrt{(12 - p)^2 + (0 - p)^2})^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p - 16)(p + 4) = 0$ $\therefore p = 16 \quad \text{or} \quad p = -4 \text{ (n.a.)}$ $\therefore T(16; 16)$	<p>✓ substitution of E & T</p> <p>✓ equating</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ $p = 16$</p> <p style="text-align: right;">(5)</p>
3.4.2a	$(x - 12)^2 + y^2 = (4\sqrt{17})^2 = 272$	<p>✓ LHS ✓ RHS</p> <p style="text-align: right;">(2)</p>
3.4.2b	$m_{\text{radius}} = \frac{1}{4}$ $m_{\text{tangent}} = -4$ $y = -4x + c$ $-4 = -4(-4) + c$ $c = -20$ $y = -4x - 20$ <p style="text-align: center;">OR/OF</p> $y - y_1 = -4(x - x_1)$ $y - (-4) = -4(x - (-4))$ $y = -4x - 20$	<p>✓ m_{tangent}</p> <p>✓ substitution of B</p> <p>✓ equation</p> <p style="text-align: right;">(3)</p>
		[24]

QUESTION/VRAAG 4



4.1	Radius = 4 units/ <i>eenhede</i>	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1; 7)$	✓ x value ✓ y value (2)
4.2.2	$CD = 6$ units $\therefore D(5; 7)$	✓ x value ✓ y value (2)
4.2.3	$\perp h = 5$ units $DC = 6$ units $\text{Area } \triangle BCD = \frac{1}{2}(6)(5)$ $= 15 \text{ units}^2$ OR/OF $\perp h = 5$ units $DC = 6$ units $\text{Area } \triangle BCD = \frac{1}{2}[\text{Area of } \parallel^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15 \text{ units}^2$	✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5$ units ✓ substitution into Area formula ✓ answer (3)

OR/OF



NAMF is a square ($NA=NF=AM=MF$ and $NA \perp AM$)

Midpoint $NM = (1; 1)$
 = Midpoint of AF

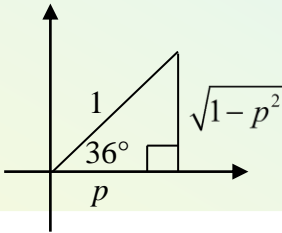
✓ NAMF = square

✓ x ✓ y of midpt NM
 ✓ midpt AF

(4)

[15]

QUESTION/VRAAG 5

5.1	$\frac{\sin 140^\circ \cdot \sin(360^\circ - x)}{\cos 50^\circ \cdot \tan(-x)}$ $= \frac{\sin 40^\circ (-\sin x)}{\sin 40^\circ (-\tan x)}$ $= \frac{-\sin x}{-\tan x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	<p>✓ $\sin 40^\circ$ ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>✓ answer</p>
5.2	$\text{LHS} = \frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540^\circ - x)}$ $\text{LHS} = \frac{-2(1 - \cos^2 x) + \cos x + 1}{1 - (-\cos x)}$ $\text{LHS} = \frac{-2 + 2\cos^2 x + \cos x + 1}{1 + \cos x}$ $\text{LHS} = \frac{2\cos^2 x + \cos x - 1}{1 + \cos x}$ $\text{LHS} = \frac{(2\cos x - 1)(\cos x + 1)}{1 + \cos x}$ $\text{LHS} = 2\cos x - 1$ $\therefore \text{LHS} = \text{RHS}$	<p>RHS = $2\cos x - 1$</p> <p>✓ identity i. t. o. $\cos x$ ✓ $\cos(540^\circ - x) = -\cos x$</p> <p>✓ standard form</p> <p>✓ factors</p>
5.3.1	$\sin 36^\circ = \sqrt{1 - p^2}$ $\tan 36^\circ = \frac{\sqrt{1 - p^2}}{p}$ <p>OR/OF</p> $\cos^2 36^\circ = 1 - \sin^2 36^\circ$ $\cos 36^\circ = \sqrt{1 - (1 - p^2)}$ $= p$ $\tan 36^\circ = \frac{\sin 36^\circ}{\cos 36^\circ}$ $= \frac{\sqrt{1 - p^2}}{p}$	 <p>✓ method ✓ value of p ✓ answer</p> <p>✓ method ✓ $\cos 36^\circ = p$</p> <p>✓ answer</p>

5.3.2	<p> $\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(2 \cos^2 36^\circ - 1)$ $= -2p^2 + 1$ </p> <p>OR/OF</p> <p> $\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(1 - 2 \sin^2 36^\circ)$ $= -1 + 2(\sqrt{1 - p^2})^2$ $= -1 + 2(1 - p^2)$ $= -2p^2 + 1$ </p> <p>OR/OF</p> <p> $\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(\cos^2 36^\circ - \sin^2 36^\circ)$ $= -\left(p^2 - (\sqrt{1 - p^2})^2\right)$ $= -(p^2 - (1 - p^2))$ $= -2p^2 + 1$ </p> <p>OR/OF</p> <p> $\cos 108^\circ$ $= \cos(2 \times 54^\circ)$ $= 2 \cos^2 54^\circ - 1$ $= 2(1 - p^2) - 1$ $= 1 - 2p^2$ </p> <p>OR/OF</p> <p> $\cos 108^\circ = \cos(72^\circ + 36^\circ)$ $= \cos 72^\circ \cos 36^\circ - \sin 72^\circ \sin 36^\circ$ $= (2 \cos^2 36^\circ - 1) \cos 36^\circ - (2 \sin 36^\circ \cos 36^\circ) \sin 36^\circ$ $= 2 \cos^3 36^\circ - \cos 36^\circ - 2 \cos 36^\circ \sin^2 36^\circ$ $= 2p^3 - p - 2p(\sqrt{1 - p^2})^2$ $= 2p^3 - p - 2p + 2p^3$ $= 4p^3 - 3p$ </p>	<p> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) </p> <p> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) </p> <p> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) </p> <p> ✓ double angle ✓✓ expansion ✓ answer i. t. o. p (4) </p> <p> ✓ expansion ✓ both double angle identities ✓ value of $\sin 36^\circ$ ✓ answer i. t. o. p (4) </p>
		[17]

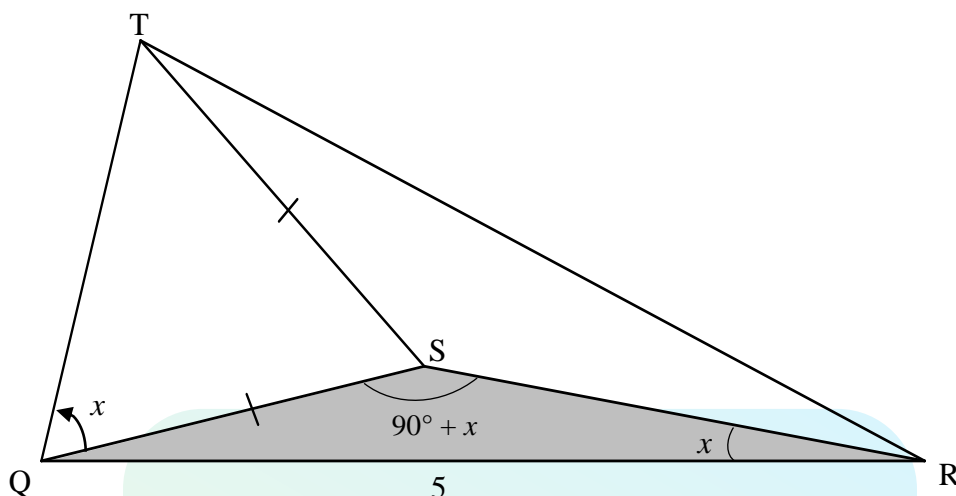
QUESTION/VRAAG 6

6.1.1	$\begin{aligned}\cos(\alpha + \beta) &= \cos(\alpha - (-\beta)) \\ &= \cos \alpha \cos(-\beta) + \sin \alpha \sin(-\beta) \\ &= \cos \alpha \cos \beta + \sin \alpha (-\sin \beta) \\ &= \cos \alpha \cos \beta - \sin \alpha \sin \beta\end{aligned}$	$\checkmark \cos(\alpha - (-\beta))$ \checkmark expansion \checkmark reduction (3)
6.1.2	$\begin{aligned}2 \cos 6x \cos 4x - \cos 10x + 2 \sin^2 x &= 2 \cos 6x \cos 4x - \cos(6x + 4x) + 2 \sin^2 x \\ &= 2 \cos 6x \cos 4x - (\cos 6x \cos 4x - \sin 6x \sin 4x) + 2 \sin^2 x \\ &= \cos 6x \cos 4x + \sin 6x \sin 4x + 2 \sin^2 x \\ &= \cos 2x + 2 \sin^2 x \\ &= 1 - 2 \sin^2 x + 2 \sin^2 x \\ &= 1\end{aligned}$	$\checkmark \cos 10x = \cos(6x + 4x)$ \checkmark expansion of $\cos(6x + 4x)$ $\checkmark \cos 2x$ $\checkmark 1 - 2 \sin^2 x$ \checkmark answer (5)
6.2	$\begin{aligned}\tan x &= 2 \sin 2x \\ \frac{\sin x}{\cos x} &= 2(2 \sin x \cos x) \\ \sin x &= 4 \sin x \cos^2 x \\ 4 \sin x \cos^2 x - \sin x &= 0 \\ \sin x(4 \cos^2 x - 1) &= 0 \\ \sin x &= 0 & \text{ or } & \cos^2 x = \frac{1}{4} \\ & & & \cos x = -\frac{1}{2} \\ x = 180^\circ + k.360^\circ; k \in \mathbb{Z} & \text{ or } & x = 120^\circ + k.360^\circ; k \in \mathbb{Z} \\ & & & x = 240^\circ + k.360^\circ; k \in \mathbb{Z}\end{aligned}$ <p>OR/OF</p> $\begin{aligned}\tan x &= 2 \sin 2x \\ \frac{\sin x}{\cos x} &= 4 \sin x \cos x \\ \sin x &= 4 \sin x \cos^2 x \\ 4 \sin x \cos^2 x - \sin x &= 0 \\ 4 \sin x(1 - \sin^2 x) - \sin x &= 0 \\ 3 \sin x - 4 \sin^3 x &= 0 \\ \sin x(3 - 4 \sin^2 x) &= 0 \\ \sin x &= 0 & \text{ or } & \sin^2 x = \frac{3}{4} \\ & & & \sin x = \frac{\sqrt{3}}{2} & \text{ or } & \sin x = -\frac{\sqrt{3}}{2} \\ x = 180^\circ + k.360^\circ, k \in \mathbb{Z} & \text{ or } & x = 120^\circ + k.360^\circ, k \in \mathbb{Z} \\ & & & \text{ or } & x = 240^\circ + k.360^\circ, k \in \mathbb{Z}\end{aligned}$	\checkmark quotient identity \checkmark double angle identity \checkmark factors \checkmark both equations $\checkmark x = 180^\circ$ $\checkmark x = 120^\circ \& 240^\circ$ OR/OF $x = \pm 120^\circ$ $\checkmark k.360^\circ; k \in \mathbb{Z}$ (7)
		(7)
		[15]

QUESTION/VRAAG 7

7.1		<ul style="list-style-type: none"> ✓ both turning points ✓ both x intercepts (-30° & 150°) ✓ shape <p>(3)</p>
7.2	Period = 120°	<ul style="list-style-type: none"> ✓✓ answer <p>(2)</p>
7.3	$x = -30^\circ$	<ul style="list-style-type: none"> ✓ answer <p>(1)</p>
7.4	<p>Range of/waardeversameling van g: $y \in [-1; 1]$</p> <p>Range of/Waardeversameling van $\frac{1}{2}g$: $y \in \left[-\frac{1}{2}; \frac{1}{2}\right]$</p> <p>Range of/Waardeversameling van $\frac{1}{2}g + 1$: $y \in \left[\frac{1}{2}; \frac{3}{2}\right]$</p> <p>OR/OF</p> <p>Range of/Waardeversameling van $\frac{1}{2}g + 1$: $\frac{1}{2} \leq y \leq \frac{3}{2}$</p>	<ul style="list-style-type: none"> ✓ critical values ✓ correct notation <p>(2)</p> <ul style="list-style-type: none"> ✓ critical values ✓ correct notation <p>(2)</p>
[8]		

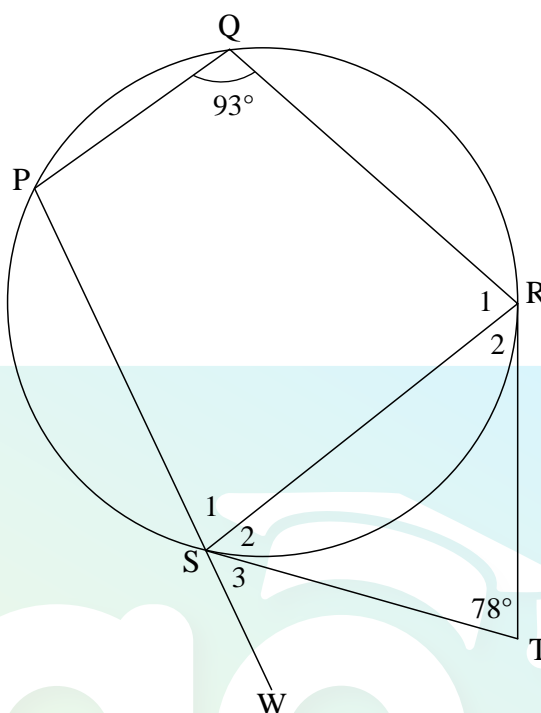
QUESTION/VRAAG 8



8.1	<p>In ΔSQR:</p> $\frac{QS}{\sin x} = \frac{QR}{\sin(90^\circ + x)}$ $\frac{QS}{\sin x} = \frac{5}{\cos x}$ $QS = \frac{5 \sin x}{\cos x}$ $QS = 5 \tan x$	<p>✓ correct use of sine rule</p> <p>✓ $\sin(90^\circ + x) = \cos x$</p> <p>✓ $QS = \frac{5 \sin x}{\cos x}$</p> <p>(3)</p>
8.2	$\frac{QT}{\sin(180^\circ - 2x)} = \frac{TS}{\sin x}$ $\frac{QT}{\sin 2x} = \frac{5 \tan x}{\sin x}$ $QT = \frac{5 \tan x \sin 2x}{\sin x}$ $QT = \frac{5 \left(\frac{\sin x}{\cos x} \right) (2 \sin x \cos x)}{\sin x}$ $QT = \frac{5 \sin x (2 \sin x)}{\sin x}$ $QT = 10 \sin x$	<p>✓ correct use of sine rule</p> <p>✓ $TS = QS = 5 \tan x$</p> <p>✓ $QT = \frac{5 \tan x \sin 2x}{\sin x}$</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>✓ $\sin 2x = 2 \sin x \cos x$</p> <p>(5)</p>

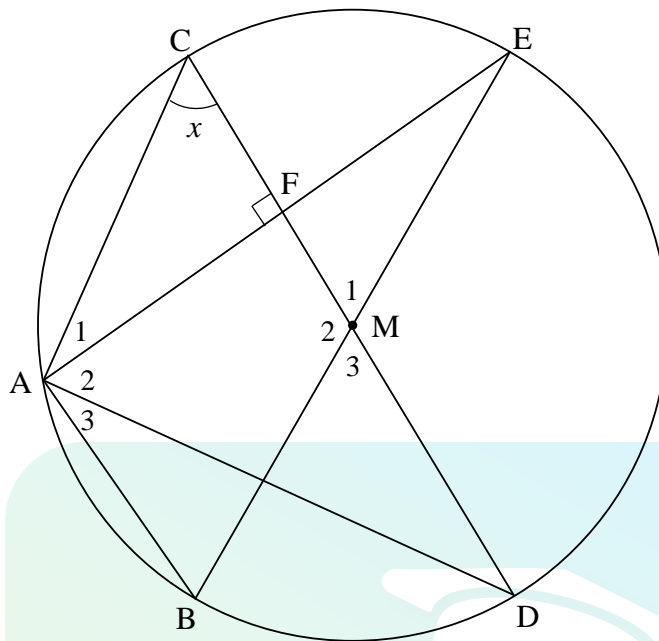
	<p>OR/OF</p> $QT^2 = QS^2 + TS^2 - 2QS.TS\cos\hat{QST}$ $QT^2 = (5 \tan x)^2 + (5 \tan x)^2 - 2(5 \tan x).(5 \tan x)\cos(180^\circ - 2x)$ $QT^2 = 50 \tan^2 x - 50 \tan^2 x(-\cos 2x)$ $QT^2 = 50 \tan^2 x(1 + \cos 2x)$ $QT^2 = 50 \tan^2 x(1 + 2 \cos^2 x - 1)$ $QT^2 = 50 \tan^2 x(2 \cos^2 x)$ $QT^2 = 100 \frac{\sin^2 x}{\cos^2 x} (\cos^2 x)$ $QT^2 = 100 \sin^2 x$ $QT = 10 \sin x$ <p>OR/OF</p> $TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5 \tan x)^2 = (5 \tan x)^2 + TQ^2 - 2(5 \tan x).TQ.\cos x$ $0 = TQ^2 - 2(5 \tan x).TQ.\cos x$ $0 = TQ[TQ - 10 \tan x.\cos x]$ $TQ = 10 \tan x.\cos x \quad (TQ \neq 0)$ $= 10 \frac{\sin x}{\cos x} . \cos x$ $= 10 \sin x$	<p>✓ correct use of cos rule ✓ $TS = QS = 5 \tan x$</p> <p>✓ $\cos 2x = 2 \cos^2 x - 1$ & reduction</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $QT^2 = 100 \sin^2 x$</p> <p>(5)</p> <p>✓ correct use of cos rule ✓ $TS = QS = 5 \tan x$ ✓ quadratic equation into TQ</p> <p>✓ $TQ = 10 \tan x . \cos x$ ✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>(5)</p>
8.3	<p>Area of $\Delta TQR = \frac{1}{2} . TQ . QR \sin \hat{TQR}$</p> $= \frac{1}{2} (10 \sin 25^\circ)(5)(\sin 70^\circ)$ $= 9,93 \text{ unit}^2$	<p>✓ correct substitution into the area rule ✓ answer</p> <p>(2)</p>
[10]		

QUESTION/VRAAG 9



9.1	tangents from same(common) point/ <i>raaklyne vanaf dieselfde punt</i>	✓ R (1)
9.2.1	$\hat{S}_2 = \hat{S}_T$ [∠s opp equal sides/∠e teenoor gelyke sye] $\therefore \hat{S}_2 = 51^\circ$ [sum of ∠s in Δ/som van ∠e in Δ]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93^\circ$ [ext ∠ of cyclic quad/ <i>buite ∠ van koordevh</i>] $\hat{S}_3 = 42^\circ$ OR/OF $\hat{S}_1 = 87^\circ$ [opp ∠s of cyclic quad/ <i>teenoorst ∠e v kdvh</i>] $\hat{S}_3 = 180^\circ - (87^\circ + 51^\circ)$ $\hat{S}_3 = 42^\circ$ [∠s on a str line/∠e op reguitlyn]	✓ R ✓ answer (2) ✓ R ✓ answer (2)
[5]		

QUESTION/VRAAG 10

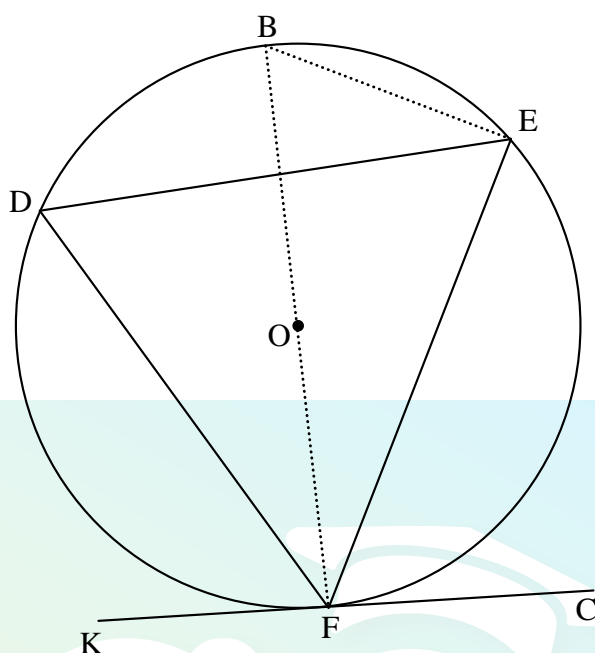


10.1	line from centre \perp to chord/ <i>lyn vanaf middelpunt \perp op koord</i>	✓ R (1)
10.2	$\therefore \hat{A}_1 = 90^\circ - x$ [sum of \angle s in Δ /som van \angle e in Δ] $\therefore \hat{M}_1 = 180^\circ - 2x$ [\angle at centre = $2 \times$ at circumf/midpts $\angle = 2 \times$ omtreks \angle]	✓ S ✓ S ✓ R (3)
10.3	$\hat{CAD} = 90^\circ$ [\angle in semi circle/ \angle in halfsirkel] $\hat{A}_2 = 90^\circ - (90^\circ - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/ <i>omgek rkl-kd st.</i>] OR/OF $\hat{EMD} = 2x$ [adj suppl \angle s/aanligg suppl \angle e] $\therefore \hat{A}_2 = x$ [\angle at centre = $2 \times \angle$ at circumf/midpts $\angle = 2 \times$ omtreks \angle] $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/ <i>omgek rkl-kd st.</i>] OR/OF $\hat{M}_3 = 180^\circ - 2x$ [vert. opp/ regoorstaande \angle e] $\therefore \hat{A}_3 = 90^\circ - x$ [\angle at centre = $2 \times \angle$ at circumf/midpts $\angle = 2 \times$ omtreks \angle] $\hat{BAE} = 90^\circ$ [\angle in semi-circle/ \angle in halfsirkel] $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/ <i>omgek rkl-kd st.</i>] OR/OF	✓ S ✓ R ✓ S ✓ R ✓ S ✓ S ✓ R ✓ R ✓ S ✓ R ✓ R ✓ R ✓ R (4)

	<p> $CD \parallel AB$ [midpt. Thm/ <i>middelpuntst.</i>] $\hat{BAE} = 90^\circ$ [\angle in semi-circle/\angle in <i>halfsirkel</i>] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.\angles; $CD \parallel AB$/verwiss \anglee] $\therefore \hat{A}_2 = x = C$ $\therefore AD$ is a tangent [converse tan-chord theorem/<i>omgek rkl-kd st.</i>] </p> <p>OR/OF</p> <p> $\hat{CAD} = 90^\circ$ [\angle in semi circle/\angle in <i>halfsirkel</i>] AC = diameter [converse \angle in semi circle/<i>omgek \angle in halfsirkel</i>] $\therefore AD$ is a tangent [converse radius \perp tangent/<i>omgek radius \perp rkl</i>] </p>	<p>✓ S ✓ R</p> <p>✓ S ✓ R</p> <p>(4)</p> <p>✓ S ✓ R ✓ S ✓ R</p> <p>(4)</p>
10.4	<p> $AF = FE$ and $BM = ME$ [given & radii] $\therefore FM = \frac{1}{2} AB = 12$ units [Midpt Theorem/<i>middelpuntstelling</i>] $EM = MB = CM = 18$ units [radii] $\therefore EB = 36$ units [diameter = 2 radius] $\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras] $AE = 12\sqrt{5}$ or 26,83 units </p> <p>OR/OF</p> <p> $AF = FE$ and $BM = ME$ [given & radii] $\therefore FM = \frac{1}{2} AB = 12$ units [Midpt Theorem/<i>middelpuntstelling</i>] $EM = MB = CM = 18$ units [radii] $\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras] $FE = 6\sqrt{5}$ $AE = 12\sqrt{5}$ or 26,83 units </p>	<p>✓ FM = 12 ✓ R</p> <p>✓ EB = 36 ✓ using Pyth correctly ✓ answer</p> <p>(5)</p> <p>✓ FM = 12 ✓ R</p> <p>✓ EM = 18 ✓ using Pyth correctly</p> <p>✓ answer</p> <p>(5)</p>
		[13]

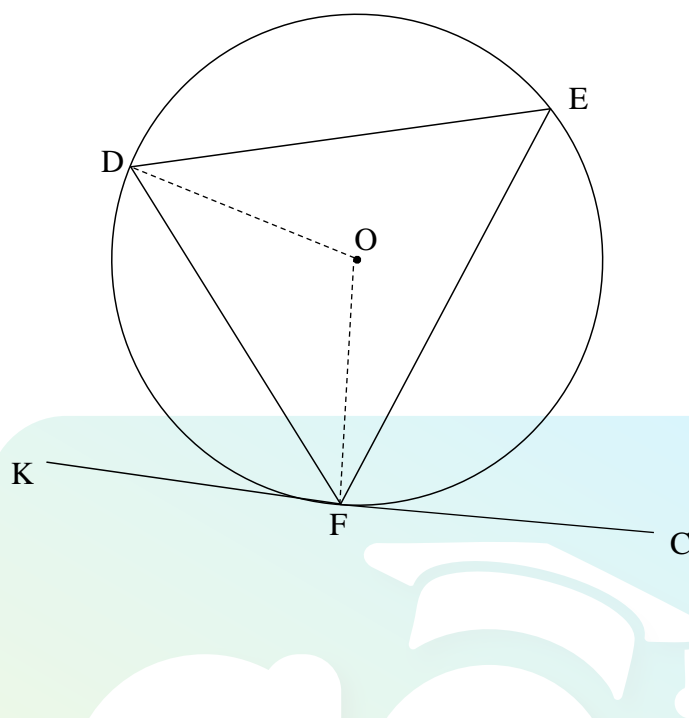
QUESTION/VRAAG 11

11.1



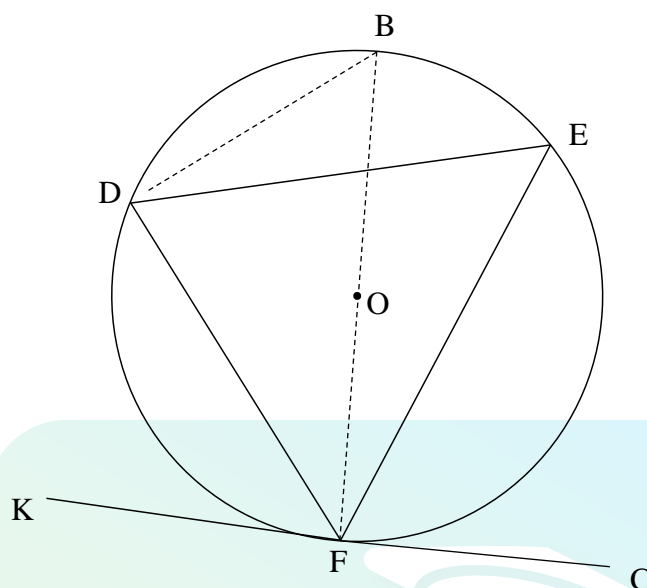
	<p>Construction: Draw diameter BF and draw BE <i>Konstruksie: Trek middellyn BF en verbind BE</i></p> <p>$\hat{B}FK = 90^\circ$ or $\hat{D}FK = 90^\circ - \hat{B}FD$ [radius \perp tangent/raaklyn]</p> <p>$\hat{B}EF = 90^\circ$ [\angle in semi-circle/semi-sirkel]</p> <p>$\therefore \hat{D}EF = 90^\circ - \hat{B}ED$</p> <p>$= 90^\circ - \hat{B}FD$ [\angles same segment/\anglee dieselfde segment]</p> <p>$\therefore \hat{D}FK = \hat{D}EF$</p>	<p>✓ Constr</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
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OR/OF



	<p>Construction: Draw radii DO and OF</p> <p><i>Konstruksie: Trek radii DO en OF</i></p> <p>$\hat{O}FK = 90^\circ$ or $\hat{D}FK = 90^\circ - \hat{O}FD$ [radius \perp tangent/raaklyn] $\hat{O}DF = \hat{O}FD$ [Angles opposite sides/∠e teenoor = sye]</p> <p>$\therefore \hat{D}OF = 180^\circ - 2\hat{O}FD$ [Angles of Δ/∠e van Δ]</p> <p>$\hat{D}EF = 90^\circ - \hat{O}FD$ [Angle at centre = $2 \times$ angle circumference/ midpoints $\angle = 2 \times$ omtreks \angle]</p> <p>$\therefore \hat{D}FK = \hat{D}EF$</p>	<p>✓ construction</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
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OR/OF



	<p>Construction: Draw diameter BF and join BD.</p> <p><i>Konstruksie: Trek middellyn BF en verbind BD.</i></p> <p>$\hat{B}FK = 90^\circ$ or $\hat{D}FK = 90^\circ - \hat{B}FD$ [radius \perp tangent/raaklyn]</p> <p>$\hat{F}DB = 90^\circ$ [\angle in half circle/semi-sirkel]</p> <p>$\hat{B} = 90^\circ - \hat{B}FD$</p> <p>$\therefore \hat{D}FK = \hat{B}$</p> <p>but $\hat{B} = \hat{E}$ [\angles same segment/\anglee dieselfde segment]</p> <p>$\therefore \hat{D}FK = \hat{E}$</p>	<p>✓ construction</p> <p>✓ S ✓/R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
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	$NKL = 180^\circ - K_4$ [adj. suppl.] $\therefore NKL = 180^\circ - NML$ [proved] $\therefore KLMN$ is a cyclic quad [opp. \angle s supplementary]	\checkmark R (1)
11.2.2	<p>In $\triangle LKN \parallel \triangle KSM$:</p> $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $\hat{L}_1 = \hat{M}_2$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $= \hat{K}_2$ [alt \angle s; / verw \angle e; $MN \parallel KS$] $N\hat{K}L = M\hat{S}K$ [\angle s of \triangle / \angle e van \triangle] $\triangle LKN \parallel \triangle KSM$ <p>OR/OF In $\triangle LKN \parallel \triangle KSM$:</p> $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $N\hat{K}L = \hat{M}_1$ [ext \angle of cyclic quad/buite \angle van koordevh] $= \hat{S}_2$ [corresp \angle s/ooreenk \angle e; $KS \parallel NM$] $\triangle LKN \parallel \triangle KSM$ [\angle , \angle , \angle] <p>OR/OF In $\triangle LKN \parallel \triangle KSM$:</p> $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [\angle s on straight line / \angle e op reguitlyn] $\therefore N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\triangle LKN \parallel \triangle KSM$ [\angle , \angle , \angle]	\checkmark S \checkmark R \checkmark S \checkmark S/R \checkmark S (5) \checkmark S \checkmark R \checkmark S/R \checkmark S \checkmark R (5)
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\triangle LKN \parallel \triangle KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units	\checkmark S \checkmark R \checkmark substitution \checkmark answer (4)
11.2.4	$4SM = 3KN$ $SM = \frac{3(8)}{4}$ $SM = 6$ $\frac{LT}{NL} = \frac{LS}{ML}$ [line \parallel one side of \triangle / lyn \parallel een sy v \triangle] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10,95$	\checkmark SM = 6 \checkmark S \checkmark R \checkmark answer (4)
		[23]

TOTAL/TOTAAL: 150